

More on [entropy function formalism for non-extremal branes](#)

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JHEP03(2009)026

(<http://iopscience.iop.org/1126-6708/2009/03/026>)

[The Table of Contents](#) and [more related content](#) is available

Download details:

IP Address: 80.92.225.132

The article was downloaded on 03/04/2010 at 10:41

Please note that [terms and conditions apply](#).

More on entropy function formalism for non-extremal branes

Ahmad Ghodsi, Mohammad R. Garousi, Tooraj Houri and Mehran Khosrav

Department of Physics, Ferdowsi University of Mashhad,

P.O. Box 1436, Mashhad, Iran

School of Physics, Institute for research in fundamental sciences (IPM),

P.O. Box 19395-5531, Tehran, Iran

E-mail: ahmad@mail.ipm.ir, garousi@mail.ipm.ir, t.houri@gmail.com,
mehran.khosravi@gmail.com

ABSTRACT: We find R^4 correction to the non-extremal $D1D5P$ solution of the supergravity by exactly solving the differential equations of motion and by using the entropy function formalism. In both cases, we find the same correction to the supergravity solution. We then calculate the correction to the entropy using the free energy method and the entropy function formalism. The results are the same.

KEYWORDS: Black Holes in String Theory, D-branes

ARXIV EPRINT: [0812.4204](https://arxiv.org/abs/0812.4204)

Contents

| | | |
|----------|---|----------|
| 1 | Introduction | 1 |
| 2 | Review of non-extremal $D1D5P$ solution | 2 |
| 3 | R^4 correction | 4 |
| 3.1 | Correction via solving differential E.O.M. | 4 |
| 3.2 | Correction via entropy function formalism | 6 |
| 4 | Entropy of nonextremal $D1D5P$ system | 7 |
| 4.1 | Entropy from free energy | 7 |
| 4.2 | Entropy from entropy function | 9 |

1 Introduction

One way of calculating the entropy of a black hole in higher derivative gravity is through the Wald formula [1]. Recently, it has been proposed by A. Sen that the Wald formula for a specific class of extremal black holes in higher derivative gravity can be written in terms of the entropy function [2]. The entropy function for the extremal black holes that their near horizon is $AdS_2 \times S^{D-2}$ is defined by integrating the Lagrangian density over S^{D-2} for a general $AdS_2 \times S^{D-2}$ background characterized by the sizes of AdS_2 and S^{D-2} , and taking the Legendre transform of the resulting function with respect to the parameters labeling the electric fields. The result is a function of moduli scalar fields as well as the sizes of AdS_2 and S^{D-2} . The values of the moduli fields and the sizes at near horizon are determined by extremizing the entropy function with respect to these fields. The entropy is then given by the value of the entropy function at its extremum¹. Using this method the near horizon solution and the entropy of some extremal black holes in the presence of higher derivative terms have been found in [2, 4, 5].

The horizon in the extremal black hole that its near horizon solution has symmetry of $AdS_2 \times S^{D-2}$ is an attractor, i.e., the physical distance between an arbitrary point and the horizon is infinite [6]. In this case, the values of scalar fields at the near horizon are independent of the values of these fields at infinity. Hence, one expects the near horizon values of these fields to be given by some algebraic equations, i.e., the equations that one

¹In above discussion, it has been assumed that in the presence of higher derivative terms the near horizon geometry has the symmetry of $AdS_2 \times S^{D-2}$. In the cases that the higher derivative corrections change this symmetry, the near horizon solution can not be found by extremizing the entropy function and the Wald formula can not be written in terms of the entropy function. In those cases one should solve the differential equations of motion to find the near horizon solution and then use the free energy method [3] or the Wald formula [1] to calculate the entropy.

finds by extremizing the entropy function [2]. We will show in this paper that in some non-extremal (near extremal) cases, even though the throat is not infinite, the near horizon values of the scalar fields are independent of the values of these fields at infinity, i.e., they are given by some algebraic equations.

It has been shown in [7] that the entropy function has a saddle point at the near horizon of extremal black holes. This may indicate that the entropy function formalism should not be specific to the extremal black holes. In fact, it has been shown in [7–9] that the entropy function formalism works for non-extremal black hole/branes at the supergravity level. However, the higher derivative terms in many cases change the symmetry of the tree level solutions, so one can not find the near horizon solution in these cases using the entropy function formalism, as in the extremal cases.

In this paper we would like to show that the higher derivative correction to the near horizon solution of the non-extremal (near extremal) $D1D5P$ can be calculated using the entropy function formalism and the Wald formula can be written in terms of the entropy function. We do this by explicitly solving the differential equations of motion and comparing the result with the near horizon solution that one finds using the entropy function formalism. Moreover, we will show that the entropy that one finds from the free energy method is the same as the entropy that one finds by equating the Wald formula with the entropy function, as in the extremal cases.

An outline of the paper is as follows. In section 2, we review the construction of near horizon solution of non-extremal $D1D5P$. In sections 3, we add the higher derivative R^4 terms to the supergravity. We find the higher derivative correction to the near horizon solution by explicitly solving the differential equations of motion in section 3.1. In section 3.2, we find the same near horizon solution using the entropy function formalism. In section 4, we study the entropy of $D1D5P$ system. In section 4.1, we calculate the entropy using the free energy method, and in section 4.2, we calculate the entropy by equating the Wald formula with the entropy function at its extremum. The results in both cases are the same.

2 Review of non-extremal $D1D5P$ solution

In this section we review the non-extremal $D1D5P$ solution of the effective action of type II string theory. The two-derivatives effective action in the string frame is given by

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left(R + 4(\partial\phi)^2 - \frac{1}{12} H_{(3)}^2 \right) - \frac{1}{2} \sum \frac{1}{n!} F_{(n)}^2 + \dots \right\}, \quad (2.1)$$

where ϕ is the dilaton, $H_{(3)}$ is NS-NS 3-form field strength, and $F_{(n)}$ is the electric R-R n -form field strength where $n = 1, 3, 5$ for IIB and $n = 2, 4$ for type IIA theory. In above equation, dots represent Fermionic terms in which we are not interested. The effective action includes a Chern-Simons term which is zero for the $D1D5P$ solution. Moreover, for this solution $F_{(n)} = dC_{(n-1)}$. The 5-form field strength tensor is self-dual, hence, it is not described by the above simple action. It is sufficient to adopt the above action for deriving the equations of motion, and impose the self-duality by hand.

The non-extremal $D1D5P$ solution of the IIB effective action when $D1$ -branes are along the compact (z) direction (S^1), $D5$ -branes are along the compact (z, x_1, x_2, x_3, x_4)

directions ($S^1 \times T^4$), the KK momentum $P = N/R$ is along the (z) direction, and the non-compact directions are (r, θ, ϕ, ψ) , is given by the following, (see e.g. [10]):

$$\begin{aligned}
 ds_{10}^2 &= (f_1 f_5)^{-\frac{1}{2}} \left(-dt^2 + dz^2 + K(\cosh \alpha_m dt - \sinh \alpha_m dz)^2 \right) \\
 &\quad + f_1^{\frac{1}{2}} f_5^{-\frac{1}{2}} \sum_{i=1}^4 dx_i^2 + (f_1 f_5)^{\frac{1}{2}} \left(\frac{dr^2}{1-K} + r^2 d\Omega_3^2 \right), \\
 e^{-2\phi} &= f_1^{-1} f_5, \\
 C_{tz} &= \coth \alpha_1 \left(\frac{1}{f_1} - 1 \right) + \tanh \alpha_1, \quad C_{tx_1 \dots x_4} = \coth \alpha_5 \left(\frac{1}{f_5} - 1 \right) + \tanh \alpha_5,
 \end{aligned}$$

where we have set the string coupling at infinity to be $g_s = 1$. In above,

$$K(r) = \frac{r_H^2}{r^2}, \quad f_1(r) = 1 + \frac{r_H^2 \sinh^2 \alpha_1}{r^2}, \quad f_5(r) = 1 + \frac{r_H^2 \sinh^2 \alpha_5}{r^2}. \quad (2.2)$$

The three conserved charges are

$$Q_1 = \frac{V r_H^2 \sinh(2\alpha_1)}{2}, \quad Q_5 = \frac{r_H^2 \sinh(2\alpha_5)}{2}, \quad N = \frac{R^2 V r_H^2 \sinh(2\alpha_m)}{2}. \quad (2.3)$$

The near horizon solution can be found by taking the following limit:

$$r^2 \ll r_H^2 \sinh^2 \alpha_{1,5}, \quad (2.4)$$

In this limit α_1 and α_5 are very large so $\sinh \alpha_{1,5} \approx \cosh \alpha_{1,5}$. In terms of the new coordinate

$$\rho^2 \equiv r^2 + r_H^2 \sinh^2 \alpha_m, \quad (2.5)$$

the near horizon solution is

$$\begin{aligned}
 ds^2 &= -\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\lambda^2 \rho^2} dt^2 + \frac{\lambda^2 \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2 \\
 &\quad + \frac{\rho^2}{\lambda^2} \left(dz - \frac{\rho_+ \rho_-}{\rho^2} dt \right)^2 + \lambda^2 d\Omega_3^2 + \frac{r_1}{r_5} \sum_{i=1}^4 dx_i^2, \\
 e^{-2\phi} &= \left(\frac{r_5}{r_1} \right)^2, \quad F_{t\rho z} = \frac{2\rho}{r_1^2}, \quad F_{t\rho z x_1 \dots x_4} = \frac{2\rho}{r_5^2}.
 \end{aligned} \quad (2.6)$$

where

$$r_{1,5}^2 \equiv r_H^2 \sinh^2 \alpha_{1,5}, \quad \rho_+ \equiv r_H \cosh \alpha_m, \quad \rho_- \equiv r_H \sinh \alpha_m, \quad \lambda^2 \equiv r_1 r_5. \quad (2.8)$$

The above metric is a direct product of $S^3 \times T^4$ and the BTZ black hole [11] upon rescaling the coordinates. The horizon $\rho = \rho_+$ in above solution is not attractor. The physical distance between an arbitrary point and horizon is

$$\int_{\rho_+}^{\rho} \frac{\lambda \rho d\rho}{\sqrt{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}} = \frac{\lambda}{2} \ln \left(\frac{\rho^2 - \frac{1}{2}(\rho_+^2 + \rho_-^2) + \sqrt{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}}{\frac{1}{2}(\rho_+^2 - \rho_-^2)} \right),$$

which is finite. For the extremal case which corresponds to $\rho_+ = \rho_-$, the distance is infinite. For the near extremal case in which we are interested, however, the distance can be made as large as we want by sending $\rho_+ \rightarrow \rho_-$. So one expects the asymptotic region to be decoupled from the near horizon region.

The higher derivative corrections to the supergravity action (2.1) may modify the near horizon solution (2.6). In general, they have field redefinition freedom [12, 13], so one may choose different scheme for the higher derivative terms. It has been argued in [14] that the scheme in which the corrections are written in terms of the 6-dimensional Weyl tensor, the near horizon solution (2.6) is not modified so it may be the reason behind the equality of the supergravity entropy and the entropy from counting the degrees of freedom for the non-extremal case [15]. In the scheme that R^4 corrections are written in terms of 10-dimensional Weyl tensor, however, the solution (2.6) is modified which may indicate that the corrections associated with the Ramond-Ramond field have nontrivial contribution to this solution in 10-dimensions. We will find the R^4 correction in the next section.

3 R^4 correction

In this section we are going to consider the string correction $\alpha'^3 R^4$ to the supergravity action. The correction in the scheme that gravity is written in terms of the 10-dimensional Weyl tensors is [16]

$$S = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left\{ \mathcal{L}^{\text{tree}} + e^{-2\phi} (\gamma W) \right\}, \quad (3.1)$$

where $\mathcal{L}^{\text{tree}}$ is given in (2.1), $\gamma = \frac{1}{8}\zeta(3)(\alpha')^3$ and W in terms of the Weyl tensor is

$$W = C^{hmnk} C_{pmnq} C_h{}^{rsp} C^q{}_{rsk} + \frac{1}{2} C^{hkmn} C_{pqmn} C_h{}^{rsp} C^q{}_{rsk}. \quad (3.2)$$

Using the above correction to the supergravity, one can find its effect on the non-extremal solution (2.6). This can be done by solving the differential equations of motion that we are going to do in the next section or by using the entropy function formalism that we will do in section 3.2.

3.1 Correction via solving differential E.O.M.

We are going to work in Euclidean space in this section. In order to find a solution in the presence of higher derivative terms, one should make an ansatz for the solution and then find the unknown functions in the ansatz by solving the differential equations of motion. We consider the following ansatz for the solution:

$$ds^2 = a(\rho) \left(\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\lambda^2 \rho^2} d\tau^2 + \frac{\lambda^2 \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2 + \frac{\rho^2}{\lambda^2} \left(dz - i \frac{\rho_+ \rho_-}{\rho^2} d\tau \right)^2 \right) + b(\rho) \left(\lambda^2 d\Omega_3^2 + \left(\frac{r_1}{r_5} \right) \sum_{i=1}^4 dx_i^2 \right), \quad (3.3)$$

$$e^{-2\phi} = u(\rho), \quad F_{\tau\rho z} = \frac{2i\rho}{r_1^2} \frac{a^{3/2}(\rho)}{b^{7/2}(\rho)}, \quad F_{\tau\rho z x_1 \dots x_4} = \frac{2i\rho}{r_5^2} a^{3/2}(\rho) b^{1/2}(\rho),$$

where $a(\rho)$, $b(\rho)$ and $u(\rho)$ are the scalar fields. We have assumed the RR charges are not modified by the higher derivative correction. With the above ansatz, the Euclidean action becomes

$$I = -\frac{1}{16\pi G_{10}} \int d^9x \int d\rho \left[\ell(a, a', a'', \dots, u, u', u'') + \gamma \omega(a, a', a'', \dots, u, u', u'') \right], \quad (3.4)$$

where ℓ and ω are

$$\ell = \sqrt{g} \left[u(\rho)R - \frac{1}{2} \frac{F_{(3)}^2}{3!} - \frac{1}{2} \frac{F_{(7)}^2}{7!} \right], \quad \omega = \sqrt{g}u(\rho)W, \quad (3.5)$$

and

$$\sqrt{g} = \frac{\rho \alpha^{3/2}(\rho) b^{7/2}(\rho) r_1^3}{r_5}, \quad \frac{F_{(3)}^2}{3!} = \frac{4r_5}{r_1^3 b^7(\rho)}, \quad \frac{F_{(7)}^2}{7!} = \frac{4r_5}{r_1^3 b^7(\rho)}. \quad (3.6)$$

The Euler-Lagrange equation for the scalar field $a(\rho)$ which follows from the above action is given by

$$\frac{\partial \ell}{\partial a} - \frac{d}{d\rho} \frac{\partial \ell}{\partial a'} + \frac{d^2}{d\rho^2} \frac{\partial^2 \ell}{\partial a''} = -\gamma \left(\frac{\partial \omega}{\partial a} - \frac{d}{d\rho} \frac{\partial \omega}{\partial a'} + \frac{d^2}{d\rho^2} \frac{\partial^2 \omega}{\partial a''} \right), \quad (3.7)$$

and similarly for $b(\rho)$ and $u(\rho)$. These differential equations are valid only to first order of γ , so one has to solve them perturbatively. At the zeroth order of γ , the solution is (3.3), i.e., $a = b = 1$, $u = (r_5/r_1)^2$. At the first order of γ , the solution must be in the following form:

$$a(\rho) = 1 + \gamma a_p(\rho), \quad b(\rho) = 1 + \gamma b_p(\rho), \quad u(\rho) = \left(\frac{r_5}{r_1} \right)^2 (1 + \gamma u_p(\rho)). \quad (3.8)$$

Inserting them in the differential equations of motion, one finds the following equations for the scalars a_p , b_p , u_p , respectively:

$$\begin{aligned} & \left(\rho(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2) \right) \left(\frac{1}{7} a_p'' + b_p'' + \frac{2}{7} u_p'' \right) + \left(3\rho^4 - \rho^2 \rho_+^2 - \rho^2 \rho_-^2 - \rho_+^2 \rho_-^2 \right) \\ & \quad \left(\frac{1}{7} a_p' + b_p' + \frac{2}{7} u_p' \right) - \left(\frac{3}{7} \rho^3 (a_p + 7b_p + 2u_p) \right) = -\frac{9}{28} \rho^3 Q_1^{-3/2} Q_5^{-3/2}, \\ & \left(\rho(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2) \right) \left(a_p'' + 3b_p'' + u_p'' \right) + \left(3\rho^4 - \rho^2 \rho_+^2 - \rho^2 \rho_-^2 - \rho_+^2 \rho_-^2 \right) \\ & \quad \left(a_p' + 3b_p' + u_p' \right) - \left(3\rho^3 (a_p - \frac{61}{21} b_p - \frac{2}{7} u_p) \right) = \frac{27}{28} \rho^3 Q_1^{-3/2} Q_5^{-3/2}, \\ & \left(\rho(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2) \right) \left(\frac{2}{7} a_p'' + b_p'' \right) + \left(3\rho^4 - \rho^2 \rho_+^2 - \rho^2 \rho_-^2 - \rho_+^2 \rho_-^2 \right) \\ & \quad \left(\frac{2}{7} a_p' + b_p' \right) + \left(\frac{6}{7} \rho^3 (-a_p + b_p) \right) = \frac{9}{14} \rho^3 Q_1^{-3/2} Q_5^{-3/2}. \end{aligned} \quad (3.9)$$

The above differential equations should give correction to the near horizon geometry (3.3). Similar equations have been found in [14] for the correction to the non-extremal D3-branes

and in [17] for the non-extremal M2-branes. In those cases, using the boundary condition at the horizon, one finds that the solution is a power law solution. However, the above equations have only constant solutions with the following values:

$$\begin{aligned}
 a(\rho) &= 1 - \gamma \frac{51}{32r_1^3 r_5^3}, \\
 b(\rho) &= 1 - \gamma \frac{27}{32r_1^3 r_5^3}, \\
 u(\rho) &= \left(\frac{r_5}{r_1}\right)^2 \left(1 + \gamma \frac{33}{8r_1^3 r_5^3}\right).
 \end{aligned}
 \tag{3.10}$$

This indicates that the near horizon geometry in the presence of the higher derivative terms is a direct product of $S^3 \times T^4$ and the BTZ black hole, as in the tree level. In [9], it has been shown that the tree level solution (3.3) is consistent with the entropy function formalism. The above result indicates that this consistency should be valid even in the presence of the higher derivative terms. Hence, the correction to the non-extremal solution (3.3) should be also found by using the entropy function formalism that we are going to do in the next section. We note however that there are many cases that the entropy function formalism works only at the tree level, e.g., non-extremal D3, M2, M5 solutions [7].

3.2 Correction via entropy function formalism

We are going to work in Minkowski space in this section. In order to find solution in the entropy function formalism, one should consider a general background with the same symmetry as the symmetry of the tree level solution, i.e.,

$$\begin{aligned}
 ds^2 &= v_1 \left(-\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\lambda^2 \rho^2} dt^2 + \frac{\lambda^2 \rho^2}{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)} d\rho^2 \right. \\
 &\quad \left. + \frac{\rho^2}{\lambda^2} \left(dz - \frac{\rho_+ \rho_-}{\rho^2} dt \right)^2 \right) + v_2 \left(\lambda^2 d\Omega_3^2 + \left(\frac{r_1}{r_5}\right) \sum_{i=1}^4 dx_i^2 \right), \\
 e^{-2\phi} &= u_s, \quad F_{t\rho z} = \frac{2\rho v_1^{3/2}}{r_1^2 v_2^{7/2}} = e_1, \quad F_{t\rho z x_1 \dots x_4} = \frac{2\rho v_1^{3/2} v_2^{1/2}}{r_5^2} = e_2,
 \end{aligned}
 \tag{3.11}$$

where the scalars v_1, v_2, u_s are constant. The algebraic equations from which these constant parameters can be found are given by extremizing the entropy function. The entropy function, on the other hand, is defined by taking the Legendre transform of function $f(v_1, v_2, u_s, e_1, e_2, \rho) = \int_H dx_H \sqrt{-g} L$ with respect to the electric fields e_1, e_2 , and dividing the result by ρ . That is

$$F(v_1, v_2, u_s) = \frac{1}{\rho} \left(e_1 \frac{\partial f}{\partial e_1} + e_2 \frac{\partial f}{\partial e_2} - f \right).
 \tag{3.12}$$

Using the Lagrangian in (3.1), one finds the following entropy function:

$$\begin{aligned}
 F(v_1, v_2, u_s) &= \frac{V_1 V_3 V_4}{16\pi G_{10}} \left(6u_s \frac{r_1^2}{r_5^2} v_1^{1/2} v_2^{5/2} (v_2 - v_1) + \frac{2v_1^{3/2}}{v_2^{7/2}} + 2v_1^{3/2} v_2^{1/2} \right. \\
 &\quad \left. - \gamma u_s \frac{r_1^3}{r_5} v_1^{3/2} v_2^{7/2} \frac{105v_2^4 - 60v_1^3 v_2 + 54v_1^2 v_2^2 - 60v_1 v_2^3 + 105v_1^4}{32r_1^4 r_5^4 v_1^4 v_2^4} \right).
 \end{aligned}
 \tag{3.13}$$

Considering the following perturbative solution:

$$v_1 = 1 + \gamma x, \quad v_2 = 1 + \gamma y, \quad u_s = \left(\frac{r_5}{r_1}\right)^2 (1 + \gamma z), \quad (3.14)$$

one finds the following equations

$$\begin{aligned} \frac{\partial F}{\partial u_s} = 0 &\quad \rightarrow \quad 6(x - y) = \frac{9}{2(r_1 r_5)^3}, \\ \frac{\partial F}{\partial v_1} = 0 &\quad \rightarrow \quad 28y + 4x + 8z = \frac{3}{(r_1 r_5)^3}, \\ \frac{\partial F}{\partial v_2} = 0 &\quad \rightarrow \quad -244y + 84x - 24z = -\frac{27}{(r_1 r_5)^3}. \end{aligned} \quad (3.15)$$

The solution to these consistent equations is

$$v_1 = 1 - \gamma \frac{51}{32(r_1 r_5)^3}, \quad v_2 = 1 - \gamma \frac{27}{32(r_1 r_5)^3}, \quad u_s = \left(\frac{r_5}{r_1}\right)^2 \left(1 + \gamma \frac{33}{8(r_1 r_5)^3}\right), \quad (3.16)$$

which is exactly the same as the solution in (3.10). Therefore, even though this system is non-extremal, the entropy function formalism and the differential equations of motion yield the same result for the near horizon background in the presence of the higher derivative terms. We now turn to the calculation of the entropy of this system.

4 Entropy of nonextremal *D1D5P* system

In the presence of higher derivative terms, the entropy can be calculated either from the free energy or from the Wald formula. For the systems that the entropy function formalism can be used to find the near horizon solution, e.g., the non-extremal *D1D5P* solution, the Wald formula can be written in terms of the entropy function which is an efficient way to calculate the entropy in the presence of higher derivative terms. In the next section we calculate the entropy using the free energy method [3], and in the section 4.2 we calculate the entropy from the Wald formula.

4.1 Entropy from free energy

Following [3], one can identify the free energy of the theory with the Euclidean gravitational action, I , times the temperature, T , i.e.

$$I = \beta \mathcal{F}, \quad (4.1)$$

where $\beta = \frac{1}{T}$. The calculation of the Euclidean action is divergent at large distances, ρ_{\max} , and requires a subtraction. The integral must be regulated by subtracting off its zero entropy limit, i.e.

$$\mathcal{F} = \lim_{\rho_{\max} \rightarrow \infty} \frac{I - I_0}{\beta}, \quad (4.2)$$

where I_0 is the zero entropy limit in which the periodicity of the Euclidean time is defined by β_0 . One must adjust β_0 so that the geometry at $\rho = \rho_{\max}$ is the same in the two

cases, i.e. the black hole and its zero entropy limit. This can be done by equating the circumference of the Euclidean time in two cases. Having \mathcal{F} , the entropy in terms of the free energy is then given by $S = -\frac{\partial \mathcal{F}}{\partial T}$.

Let us start from the black hole action which will be noted by I_{BH}

$$I_{\text{BH}} = -\frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{g} \mathcal{L}, \quad (4.3)$$

where \mathcal{L} is given in (3.1). Inserting the solution (3.3) in which $a(\rho)$, $b(\rho)$, $u(\rho)$ are given in (3.10), one finds

$$\begin{aligned} I_{\text{BH}} &= \frac{1}{4\pi G_{10}} \int_0^\beta d\tau \int_{V_1} dz \int_{V_3} d\Omega_3 \int_{V_4} dx_1 \dots dx_4 \int_{\rho_+}^{\rho_{\text{max}}} \left(1 - \gamma \frac{9}{8r_1^3 r_5^3} + O(\gamma^2)\right) \rho d\rho \\ &= \frac{1}{8\pi G_{10}} V_1 V_3 V_4 \beta \left(1 - \gamma \frac{9}{8r_1^3 r_5^3} + O(\gamma^2)\right) (\rho_{\text{max}}^2 - \rho_+^2), \end{aligned} \quad (4.4)$$

where ρ_{max} is a cutoff at large distances. The above expression is divergent at large distances and must be regulated by subtracting off its zero entropy limit, I_{AdS} . It is given by

$$\begin{aligned} I_{\text{AdS}} &= \frac{1}{4\pi G_{10}} \int_0^{\beta_0} d\tau \int dz \int d\Omega_3 \int dx_1 \dots dx_4 \int_0^{\rho_{\text{max}}} \left(1 - \gamma \frac{9}{8r_1^3 r_5^3} + O(\gamma^2)\right) \rho d\rho, \\ &= \frac{1}{8\pi G_{10}} V_1 V_3 V_4 \beta_0 \left(1 - \gamma \frac{9}{8r_1^3 r_5^3} + O(\gamma^2)\right) (\rho_{\text{max}}^2). \end{aligned} \quad (4.5)$$

The relation between β_0 and β is

$$\beta_0 = \beta \sqrt{\frac{(\rho^2 - \rho_+^2)(\rho^2 - \rho_-^2)}{\rho^4}} \Big|_{\rho=\rho_{\text{max}}} \cong \beta \left(1 - \frac{(\rho_+^2 + \rho_-^2)}{2\rho_{\text{max}}^2}\right), \quad (4.6)$$

which comes from the fact that the geometry of the hypersurface $\rho = \rho_{\text{max}}$ must be the same for both cases [3].

Taking the limit $\rho_{\text{max}} \rightarrow \infty$ of the subtraction of I_{BH} and I_{AdS} , one finds the free energy in terms of r_H to be

$$\mathcal{F} = \lim_{\rho_{\text{max}} \rightarrow \infty} \left(\frac{I_{\text{BH}} - I_{\text{AdS}}}{\beta} \right) = -\frac{1}{16\pi G_{10}} V_1 V_3 V_4 \left(1 - \gamma \frac{9}{8r_1^3 r_5^3} + O(\gamma^2)\right) r_H^2. \quad (4.7)$$

To write it in terms of temperature, one calculates the surface gravity by KK reduction to 9-dimension (see e.g., [18]) which has diagonal metric, i.e.,

$$\kappa = 2\pi T = \sqrt{G^{\rho\rho}} \frac{d}{d\rho} \sqrt{G_{\tau\tau}} \Big|_{\text{Horizon}}, \quad (4.8)$$

where

$$G_{\tau\tau} = g_{\tau\tau} - \frac{g_{\tau z}^2}{g_{zz}}, \quad G^{\rho\rho} = g^{\rho\rho}. \quad (4.9)$$

One finds the temperature to be

$$T = \frac{(\rho_+^2 - \rho_-^2)}{2\pi\lambda^2\rho_+} = \frac{1}{2\pi r_H \cosh(\alpha_1) \cosh(\alpha_5) \cosh(\alpha_m)},$$

Using the fact that the number of D1 and D5 branes, N_1 , N_5 , and the boost parameter α_m are independent of temperature, one finds the following linear relation between temperature and r_H :

$$T = \frac{\sqrt{V}}{2\pi\ell_s^4\sqrt{N_1N_5}\cosh\alpha_m}r_H, \tag{4.10}$$

where we have used

$$N_1 = \frac{Vr_H^2}{2l_s^6}\sinh(2\alpha_1), \quad N_5 = \frac{r_H^2}{2l_s^2}\sinh(2\alpha_5), \tag{4.11}$$

and the fact that in the near horizon region $\sinh(\alpha_{1,5}) \approx \cosh(\alpha_{1,5})$. The entropy $S = -\frac{\partial\mathcal{F}}{\partial T}$ becomes

$$S = \frac{1}{8\pi G_{10}}V_1V_3V_4\left(1 - \gamma\frac{9}{8r_1^3r_5^3} + O(\gamma^2)\right)\frac{2\pi\ell_s^4\sqrt{N_1N_5}r_H\cosh\alpha_m}{\sqrt{V}}. \tag{4.12}$$

It is convenient to write the entropy in terms of the left and right KK momenta, N_L and N_R which are defined as

$$N_{L,R} = \frac{VR_z^2r_H^2}{4l_s^8}\exp(\pm 2\alpha_m), \tag{4.13}$$

Using above relations, one finds the entropy to be

$$S = 2\pi\sqrt{N_1N_5}(\sqrt{N_L} + \sqrt{N_R})\left(1 - \gamma\frac{9}{8r_1^3r_5^3} + O(\gamma^2)\right). \tag{4.14}$$

where we have also used the relevant formulas for the volume of the circle, 3-sphere and the volume of 4-torus as well as the the 10-dimensional Newton constant, i.e.,

$$V_1 = 2\pi R_z, \quad V_3 = 2\pi^2, \quad V_4 = (2\pi)^4V, \quad G_{10} = 8\pi^6\ell_s^8, \tag{4.15}$$

The first term in (4.14) is the supergravity result (see e.g., [18]) and the second term is the higher derivative correction. In the next section we calculate the entropy using the entropy function formalism.

4.2 Entropy from entropy function

Entropy in a higher derivative theory can also be calculated from the Wald formula [1]

$$S_{\text{BH}} = 4\pi \int_H dx_H \sqrt{-g_H} \frac{\partial L}{\partial R_{\mu\nu\lambda\rho}} g_{\mu\lambda}^\perp g_{\nu\rho}^\perp, \tag{4.16}$$

where L is the Lagrangian density and $g_{\mu\nu}^\perp$ denotes the metric projection onto subspace orthogonal to the horizon. It has been shown in [2] that for extremal black holes that the

near horizon geometry can be calculated using the entropy function formalism, the Wald formula is proportional to the entropy function. We have seen that the correction to the non-extremal solution (3.3) can also be calculated using the entropy function formalism. Hence, one expects that the Wald formula in this case also is proportional to the entropy function. The constant of the proportionality can be fixed by comparing it with the entropy at the supergravity level. That is

$$S_{\text{BH}} = \frac{\pi \ell_s^4 \sqrt{N_1 N_5} \rho_+}{\sqrt{V}} F, \tag{4.17}$$

where F is the entropy function. One can easily check that the above entropy is the same as the tree level entropy (4.14) after inserting the tree level entropy function (3.13) into it. The above formula can also be found directly from the Wald formula [5, 8, 9].

Now, inserting the solution (3.16) into the entropy function (3.13), one finds the entropy function at its extremum to be

$$F = \frac{1}{8\pi G_{10}} V_1 V_3 V_4 \left(1 - \gamma \frac{9}{8r_1^3 r_5^3} + O(\gamma^2) \right) \tag{4.18}$$

After inserting this into (4.17) and using (4.13) and (4.15), one finds exactly the entropy in (4.14). This confirms that the Wald formula (4.16) for the non-extremal $D1D5P$ solution in the presence of higher derivative terms is proportional to the entropy function. For extremal case, i.e., $N_R = 0$, the entropy has been found in [5], however, the correction is different from the one in (4.14). This is related to the fact that the scheme for the higher derivative terms in [5] is different from the scheme that we have chosen in (3.2). When there is no KK momentum, i.e., $N_R = N_L$, the entropy (4.14) is the same as the entropy that has been found in [8].

We have done the same calculation for the non-extremal $D2D6NS5P$ solution and found that the correction to the tree level solution can be calculated either by solving the differential equations of motion or by using the entropy function formalism. In this case also the Wald formula is proportional to the entropy function and it is equal to the entropy that one finds using the free energy method.

The reason that the entropy function formalism works for the non-extremal (near extremal) $D1D5P$ and $D2D6NS5P$ cases may be related to the fact that the near horizon geometry of these solutions have a throat. Even though the physical length of the throat is finite for non-extremal cases, the throat can be made as long as we want for near extremal cases. Hence, the scalar fields at the horizon are independent of the values of these fields at infinity. The tree level solutions of non-extremal (near extremal) $D3$, M_2 and $M5$ at the near horizon also have throat geometries, however, the modified solutions in the presence of the higher derivative correction have no longer the throat. That is why the entropy function formalism works for these systems only at tree level. For non-extremal $D1D5P$ and $D2D6NS5P$ cases, however, the higher derivative corrections keep the tree level throat. Hence, the entropy function formalism works even in the presence of the higher derivative terms.

In fact the entropy function formalism for extremal cases [2] works in above sense. That is, the original Wald formula for black hole entropy holds for non-extremal black holes, and

in applying this result to extremal case one must define the entropy of an extremal black hole to be the limit of the entropy of the associated non-extremal black hole in which the non-extremal parameter goes to zero. For example, the entropy of extremal $D1D5P$ is given by the entropy of non-extremal $D1D5P$ (4.14) in which $N_R \rightarrow 0$. Hence, one may expect the entropy function formalism works for any near extremal solution which has throat at the near horizon region, and entropy to leading order of the non-extremal parameter can be found using the entropy function formalism.

References

- [1] R.M. Wald, *Black hole entropy is the Nöther charge*, *Phys. Rev. D* **48** (1993) 3427 [[gr-qc/9307038](#)] [[SPIRES](#)];
T. Jacobson and R.C. Myers, *Black hole entropy and higher curvature interactions*, *Phys. Rev. Lett.* **70** (1993) 3684 [[hep-th/9305016](#)] [[SPIRES](#)];
T. Jacobson, G. Kang and R.C. Myers, *Black hole entropy in higher curvature gravity*, [gr-qc/9502009](#) [[SPIRES](#)].
- [2] A. Sen, *Black hole entropy function and the attractor mechanism in higher derivative gravity*, *JHEP* **09** (2005) 038 [[hep-th/0506177](#)] [[SPIRES](#)]; *Entropy function for heterotic black holes*, *JHEP* **03** (2006) 008 [[hep-th/0508042](#)] [[SPIRES](#)].
- [3] S.W. Hawking and D.N. Page, *Thermodynamics of black holes in Anti-de Sitter space*, *Commun. Math. Phys.* **87** (1983) 577 [[SPIRES](#)];
E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [[hep-th/9803131](#)] [[SPIRES](#)].
- [4] P. Prester, *Lovelock type gravity and small black holes in heterotic string theory*, *JHEP* **02** (2006) 039 [[hep-th/0511306](#)] [[SPIRES](#)];
A. Sinha and N.V. Suryanarayana, *Extremal single-charge small black holes: entropy function analysis*, *Class. Quant. Grav.* **23** (2006) 3305 [[hep-th/0601183](#)] [[SPIRES](#)];
B. Sahoo and A. Sen, *Higher derivative corrections to non-supersymmetric extremal black holes in $N = 2$ supergravity*, *JHEP* **09** (2006) 029 [[hep-th/0603149](#)] [[SPIRES](#)];
G. Exirifard, *The α' stretched horizon in heterotic string*, *JHEP* **10** (2006) 070 [[hep-th/0604021](#)] [[SPIRES](#)];
B. Chandrasekhar, *Born-Infeld corrections to the entropy function of heterotic black holes*, *Braz. J. Phys.* **37** (2007) 349 [[hep-th/0604028](#)] [[SPIRES](#)];
R.-G. Cai and D.-W. Pang, *Entropy function for 4-charge extremal black holes in type IIA superstring theory*, *Phys. Rev. D* **74** (2006) 064031 [[hep-th/0606098](#)] [[SPIRES](#)];
A. Sinha and N.V. Suryanarayana, *Two-charge small black hole entropy: string-loops and multi-strings*, *JHEP* **10** (2006) 034 [[hep-th/0606218](#)] [[SPIRES](#)];
J.F. Morales and H. Samtleben, *Entropy function and attractors for AdS black holes*, *JHEP* **10** (2006) 074 [[hep-th/0608044](#)] [[SPIRES](#)];
B. Sahoo and A. Sen, *α' -corrections to extremal dyonic black holes in heterotic string theory*, *JHEP* **01** (2007) 010 [[hep-th/0608182](#)] [[SPIRES](#)];
G.L. Cardoso, J.M. Oberreuter and J. Perz, *Entropy function for rotating extremal black holes in very special geometry*, *JHEP* **05** (2007) 025 [[hep-th/0701176](#)] [[SPIRES](#)];
K. Goldstein and R.P. Jena, *One entropy function to rule them all*, *JHEP* **11** (2007) 049 [[hep-th/0701221](#)] [[SPIRES](#)];

- R.-G. Cai and D.-W. Pang, *On entropy function for supersymmetric black rings*, *JHEP* **04** (2007) 027 [[hep-th/0702040](#)] [[SPIRES](#)];
- M. Alishahiha, *On R^2 corrections for 5D black holes*, *JHEP* **08** (2007) 094 [[hep-th/0703099](#)] [[SPIRES](#)];
- O.J.C. Dias and P.J. Silva, *Euclidean analysis of the entropy functional formalism*, *Phys. Rev. D* **77** (2008) 084011 [[arXiv:0704.1405](#)] [[SPIRES](#)];
- M. Cvitan, P.D. Prester, S. Pallua and I. Smolic, *Extremal black holes in $D = 5$: SUSY vs. Gauss-Bonnet corrections*, *JHEP* **11** (2007) 043 [[arXiv:0706.1167](#)] [[SPIRES](#)];
- A. Sen, *Black hole entropy function, attractors and precision counting of microstates*, *Gen. Rel. Grav.* **40** (2008) 2249 [[arXiv:0708.1270](#)] [[SPIRES](#)];
- M. Cvitan, P.D. Prester and A. Ficnar, *$(\alpha')^2$ -corrections to extremal dyonic black holes in heterotic string theory*, *JHEP* **05** (2008) 063 [[arXiv:0710.3886](#)] [[SPIRES](#)];
- D. Astefanesei, H. Nastase, H. Yavartanoo and S. Yun, *Moduli flow and non-supersymmetric AdS attractors*, *JHEP* **04** (2008) 074 [[arXiv:0711.0036](#)] [[SPIRES](#)];
- R.-G. Cai, C.-M. Chen, K.-i. Maeda, N. Ohta and D.-W. Pang, *Entropy function and universality of entropy-area relation for small black holes*, *Phys. Rev. D* **77** (2008) 064030 [[arXiv:0712.4212](#)] [[SPIRES](#)];
- X.-H. Ge and F.-W. Shu, *On black hole thermodynamics and entropy function method*, [arXiv:0804.2123](#) [[SPIRES](#)];
- F.-W. Shu and X.-H. Ge, *Entropy function and higher derivative corrections to entropies in (Anti-)de Sitter space*, *JHEP* **08** (2008) 021 [[arXiv:0804.2724](#)] [[SPIRES](#)];
- D. Astefanesei, N. Banerjee and S. Dutta, *(Un)attractor black holes in higher derivative AdS gravity*, *JHEP* **11** (2008) 070 [[arXiv:0806.1334](#)] [[SPIRES](#)].
- [5] A. Ghodsi, *R^4 corrections to D1D5P black hole entropy from entropy function formalism*, *Phys. Rev. D* **74** (2006) 124026 [[hep-th/0604106](#)] [[SPIRES](#)].
- [6] R. Kallosh, N. Sivanandam and M. Soroush, *The non-BPS black hole attractor equation*, *JHEP* **03** (2006) 060 [[hep-th/0602005](#)] [[SPIRES](#)].
- [7] M.R. Garousi and A. Ghodsi, *On attractor mechanism and entropy function for non-extremal black holes/branes*, *JHEP* **05** (2007) 043 [[hep-th/0703260](#)] [[SPIRES](#)].
- [8] M.R. Garousi and A. Ghodsi, *Entropy function for non-extremal D1D5 and D2D6NS5-branes*, *JHEP* **10** (2007) 036 [[arXiv:0705.2149](#)] [[SPIRES](#)].
- [9] R.-G. Cai and D.-W. Pang, *Entropy function for non-extremal black holes in string theory*, *JHEP* **05** (2007) 023 [[hep-th/0701158](#)] [[SPIRES](#)].
- [10] K. Sfetsos and K. Skenderis, *Microscopic derivation of the Bekenstein-Hawking entropy formula for non-extremal black holes*, *Nucl. Phys. B* **517** (1998) 179 [[hep-th/9711138](#)] [[SPIRES](#)].
- [11] M. Bañados, C. Teitelboim and J. Zanelli, *The black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [[hep-th/9204099](#)] [[SPIRES](#)];
M. Bañados, M. Henneaux, C. Teitelboim and J. Zanelli, *Geometry of the $(2 + 1)$ black hole*, *Phys. Rev. D* **48** (1993) 1506 [[gr-qc/9302012](#)] [[SPIRES](#)].
- [12] A.A. Tseytlin, *Vector field effective action in the open superstring theory*, *Nucl. Phys. B* **276** (1986) 391 [Erratum *ibid.* **B 291** (1987) 876] [[SPIRES](#)].
- [13] A.A. Tseytlin, *Ambiguity in the effective action in string theories*, *Phys. Lett. B* **176** (1986) 92 [[SPIRES](#)];
R.R. Metsaev and A.A. Tseytlin, *Curvature cubed terms in string theory effective actions*,

- Phys. Lett. B* **185** (1987) 52 [SPIRES].
- [14] S.S. Gubser, I.R. Klebanov and A.A. Tseytlin, *Coupling constant dependence in the thermodynamics of $N = 4$ supersymmetric Yang-Mills theory*, *Nucl. Phys. B* **534** (1998) 202 [[hep-th/9805156](#)] [SPIRES].
- [15] C.G. Callan and J.M. Maldacena, *D-brane approach to black hole quantum mechanics*, *Nucl. Phys. B* **472** (1996) 591 [[hep-th/9602043](#)] [SPIRES];
G.T. Horowitz and A. Strominger, *Counting states of near-extremal black holes*, *Phys. Rev. Lett.* **77** (1996) 2368 [[hep-th/9602051](#)] [SPIRES].
- [16] M.T. Grisaru and D. Zanon, *σ -model superstring corrections to the Einstein-Hilbert action*, *Phys. Lett. B* **177** (1986) 347 [SPIRES];
M.D. Freeman, C.N. Pope, M.F. Sohnius and K.S. Stelle, *Higher order σ -model counterterms and the effective action for superstrings*, *Phys. Lett. B* **178** (1986) 199 [SPIRES];
Q.-H. Park and D. Zanon, *More on σ -model β -functions and low-energy effective actions*, *Phys. Rev. D* **35** (1987) 4038 [SPIRES];
D.J. Gross and E. Witten, *Superstring modifications of Einstein's equations*, *Nucl. Phys. B* **277** (1986) 1 [SPIRES].
- [17] M.R. Garousi, A. Ghodsi and M. Khosravi, *On thermodynamics of $N = 6$ superconformal Chern-Simons theories at strong coupling*, *JHEP* **08** (2008) 067 [[arXiv:0807.1478](#)] [SPIRES].
- [18] A.W. Peet, *TASI lectures on black holes in string theory*, [hep-th/0008241](#) [SPIRES].